KANYASHREE UNIVERSITY

M.Sc. 2nd Semester Examination-2024

Subject: Mathematics

Course- CC 10

Partial Differential Equations

Full Marks-40 **Time-2.00 Hours**

GROUP - A

(Answer **any four** of the following) **(5×4=20)**

1. (i) Form a partial differential equation by eliminating arbitrary constants a and b from the relation

$$z = ax + by + a^2 + b^2.$$

(ii) Using Charpit's method find a complete integral of $z = px + qy + p^2 + q^2$. 2+3

2. Solve the following:
$(D^3 - 4D^2D' + 4DD'^2)z = 2\sin(3x + 2y)$, where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$. 5

3. Find a complete integral of the following differential equation by Jacobi's method.
${p_1}^3 + {p_2}^2 + p_3 = 1$, where $p_1 = \frac{\partial z}{\partial x_1}$, $p_2 = \frac{\partial z}{\partial x_2}$ and $p_3 = \frac{\partial z}{\partial x_3}$. 5

4. Solve one-dimensional wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$$

by the method of separation of variable with the boundary conditions

$$y(0, t) = 0 = y(a, t).$$ 5

5. Find the equation of the integral surface of the differential equation

$$2y(z-3)p + (2x - z)q = y(2x - 3)$$

which passes through the circle $z = 0,\ x^2 + y^2 = 2x$. 5

6. Solve the one-dimensional heat equation $u_t = ku_{xx}$ by Lie algebraic method. 5

GROUP – B

(Answer **any two** of the following) **(10× 2=20)**

1. Reduce the equation $y^2\, z_{xx} - 2xy\, z_{xy} + x^2 z_{yy} = \frac{y^2}{x} z_x + \frac{x^2}{y} z_y$ to canonical form and hence solve it. 10

2. State Duhamel's principle for a diffusion equation. Use it to solve the IBVP

$$u_t - \alpha^2 u_{xx} = t[\sin(2\pi x) + 2x]\,, 0 \le x \le 1, 0 < t < \infty$$

$$u(0,t) = 1, u(1,t) = t^2, 0 < t < \infty$$

$$u(x,0) = 1 + \sin(\pi x) - x.$$

2+8

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3. Find the D'Alembert solution of the Cauchy problem for the one-dimensional wave equation. Hence find the solution of the initial-value problem

$$u_{tt} = c^2 u_{xx}\,, x \in \mathbb{R}, t > 0$$

$$u(x,0) = \sin x\,, u_t(x,0) = \cos x.$$

7+3

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