KANYASHREE UNIVERSITY

M.Sc. 2nd Semester Examination-2024

Subject: Mathematics

Course- CC 9

Topology

Full Marks-40 **Time-2.00 Hours**

GROUP - A

(Answer **any four** of the following) **(5×4=20)**

1. Define connected space. Prove that a topological space (X, τ) is connected if and only if no continuous function on X into the discrete two-point space $\{0,1\}$ is surjective. 1+4
2. When a Topological Space is said to be connected? Show that the image of a connected space under a continuous map is connected. 1+4
3. Let Y be a subspace of a topological space (X, τ). Prove that if Y is open in (X, τ) then a subset G of Y is open in (Y, τ_Y) iff G is open in (X, τ), the interior of a subset A of the open set Y in (Y, τ_Y) is the same as the interior of A in (X, τ). 5
4. State Lebesgue number lemma. Show that compactness is a closed hereditary property. 1+4
5. Define Hausdorff space. Prove that if $f: (X, \tau) \to (Y, \tau_1)$ is injective and continuous, where (Y, τ_1) is Hausdorff, then (X, τ) is Hausdorff. 2+3
6. Prove that every second countable space is Lindelöf. Give an example with proper justification of a first countable space which is not second countable. 2+3

GROUP – B

(Answer **any two** of the following) **(10× 2=20)**

7. i) Define Path connected space. Show that every path connected space is connected. Is the converse true? Give proper justification.
   ii) Prove that any closed subspace of a Lindelof space is Lindelof. 6+4
8. i) State Urysohn lemma for Topological space. Prove it in a Metric space.
   ii) Show that (0, 1) is not a compact set. (2+4) +4
9. i) State and prove Heine's continuity criterion.
   ii) Prove that for any subset A of a topological space (X, τ), $A^\circ = X - \overline{(X - A)}$. 5+5

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