KANYASHREE UNIVERSITY

M.Sc. 2nd Semester Examination-2024

Subject: Mathematics

Course- CC 7

Abstract Algebra

Full Marks-40 **Time-2.00 Hours**

GROUP - A

(Answer **any four** of the following) **(5×4=20)**

1. Prove that every finite group G is isomorphic to a permutation group. 5

2. Define simple group. Show that there is no simple group of order 63. 2+3

3. State and prove Sylow's first theorem on group theory. 2+3

4. Prove that every Euclidean Domain is a Principal Ideal Domain. 5

5. Show that the integral domain $\mathbf{Z}\left[i\sqrt{5}\right]$ is a factorization domain but not a unique factorization domain. 5

6. Define splitting field. Find the degree of the splitting field of x^4+1 over $\mathbf{Q}$. 2+3

GROUP – B

(Answer **any two** of the following) **(10× 2=20)**

7. i) Define solvable group. Show that every group of prime order is solvable.

   ii) Show that the number of Sylow p-subgroup of G is of the form 1+kp, where $(1+kp) \mid o(G)$, k being a non-negative integer. 5+5

8. i) State Eisenstein Criterion of irreducibility over $\mathbf{Q}$. Show that $8x^3-6x-1$ is an irreducible polynomial over $\mathbf{Q}$.

   ii) Show that 3 is an irreducible but not a prime element in $\mathbf{Z}\left[i\sqrt{5}\right]$. 5+5

9. i) If I be an ideal of a ring R and I, R/I are both left Noetherian rings, then show that R is left Noetherian.

   ii) Let K be an extension of a field F. Then show that an element $a \in K$ is algebraic over F if and only if $\left[F(a):F\right]$ is finite. 5+5

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