KANYASHREE UNIVERSITY

M.Sc. 1st Semester Examination-2024

Subject: Mathematics

Course- CC 5

Classical Mechanics

Full Marks-40 **Time-2.00 Hours**

GROUP - A

(Answer **any four** of the following) **(5×4=20)**

1. What are constraints? Classify different types of constraints.
2. (i) Suppose L' be a function derived from a Lagrangian(L) of a mechanical system such that $L' = L + \frac{dF}{dt}$ where F is a function of t and generalised co-ordinate(q_k) only. Then show that

$$\frac{d}{dt}\left(\frac{\partial L'}{\partial \dot{q}_k}\right) - \frac{\partial L'}{\partial q_k} = 0, \quad k = 1,2 \ldots, n.$$

(ii) Derive Lagrangian and then find Lagrange's equation of motions for a particle moving in two dimensional polar-coordinate system.

(3+2)

3. Show that the area of a surface of revolution of a curve $y = y(x)$ is minimum when the curve is catenary.
4. Using transversality conditions find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$.
5. A particle of mass m, acted upon by gravity only, is sliding on a wire bent in the form of a parabola $y = x^2/2$. Construct the Hamiltonian and hence write down Hamiltonian equations of motions. (3+2)
6. Define Euler's angles. Hence obtain the components of the angular momentum of a body, rotating about an axis through the origin, along body set of axes. (2+3)

GROUP – B

(Answer **any two** of the following) **(10× 2=20)**

7. (i) State D'Alembert's principle. Use it to derive Lagrange's equations of motion for holonomic, bilateral and conservative system having n degrees of freedom.

(ii) Lagrangian of a system is given by

$$L = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - V + A\dot{x} + B\dot{y} + C\dot{z}\,,$$

where A, B, C and V are functions of (x, y, z). Show that Lagrange's equations of motion are

$$\ddot{x} + \left(\frac{\partial A}{\partial y} - \frac{\partial B}{\partial x}\right)(\dot{y}) + \left(\frac{\partial A}{\partial z} - \frac{\partial C}{\partial x}\right)(\dot{z}) + \frac{\partial V}{\partial x} = 0$$ and similar equations. (6+4)

8. (i) Find the curve on which a particle will slide from one point to another point in shortest time under gravity (Friction and resistance of the medium are neglected)

(ii) Find the geodesic of a right circular cylinder of radius a. (5+5)

9. (i) Define cyclic co-ordinate. Solve by using Routhian process the following Lagrangian system defined by

$$L = T - V = \frac{1}{2}\left(\frac{\dot{q_1}^2}{a + bq_2^2} + \dot{q_2}^2\right) - c - dq_2^2$$

where a, b, c, d are constants and q_1, q_2 are generalised co-ordinates.

(ii) Let L be the Lagrangian of a free particle given in parabolic co-ordinate (ξ, η, ϕ) as $L = \frac{1}{2}m(\xi^2 + \eta^2)\left(\dot{\xi}^2 + \dot{\eta}^2\right) + \frac{1}{2}m\xi^2\eta^2\dot{\phi}^2$.

Find Hamiltonian of the system.

(6+4)

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