KANYASHREE UNIVERSITY

M.Sc. 1st Semester Examination-2024

Subject: Mathematics

Course- CC 4

Ordinary Differential Equations and Special Functions

Full Marks-40 **Time-2.00 Hours**

GROUP - A

(Answer **any four** of the following) **(5×4=20)**

1. State and prove the Abel's identity.
2. Find the general solution of the differential equation $(x^2-1)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=(x^2-1)^2,$ given that $y=x$ is a solution of the corresponding homogeneous equation.
3. State and prove Gronwall's Lemma.
4. Obtain a series solution of the differential equation $\frac{d^2w}{dz^2}-2z\frac{dw}{dz}+2nw=0$ in the neighbourhood of $z=0$, where n is a constant.
5. If $P_n(z)$ is the Legendre polynomial of degree n, then show that $$\int_{-1}^{1} P_n(z)\,P_m(z)\,dz = \begin{matrix} 0 \; if\; m\neq n \\ \frac{2}{2n+1}\; if\; m=n. \end{matrix}$$
6. Prove that $2nJ_n(z)=z[J_{n-1}(z)+J_{n+1}(z)]$ where $J_n(z)$ is the Bessel's function of first kind of order n.

GROUP – B

(Answer **any two** of the following) **(10× 2=20)**

7. i) If $y_1, y_2, \dots, y_n$ are n solutions of $y^{(n)}+p_1(x)y^{(n-1)}+\ \dots\ +p_n(x)y=0$ on an interval I, then prove that they are linearly independent there if and only if $W(y_1, y_2, \dots, y_n)\neq 0$ for all x in I.

   ii) Using the method of variation of parameter, find a particular solution of the differential equation $\frac{d^2y}{dx^2}+4y=\tan 2x$.

   iii) Show that $L_n(0)=1\ and\ L'_n(0)=-n$ where $L_n(z)$ is the Laguerre polynomial of degree n. (5+3+2)

8. i) State and prove Sturm separation theorem.

   ii) Using the Picard's method of successive approximations, find the third approximation of the solution of the initial value problem $\frac{dy}{dx}=x+y^2,\ y(0)=0.$

   iii) Show that $H'_n(z)=2nH_{n-1}(z)$ where $H_n(z)$ is the Hermite polynomial of degree n.

   (4+3+3)

9. i) Prove that the eigen functions of the regular Sturm Liouville problem

$$\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + \{q(x) + \delta r(x)\}y = 0$$

with boundary conditions

$$A_1 y(a) + B_1 y'(a) = 0$$

and $A_2 y(b) + B_2 y'(b) = 0$

are orthogonal on the interval $[a, b]$ with weight function $r(x)$.

ii) Solve the following boundary value problem by constructing Green's function:

$$\frac{d^2u}{dx^2} + u = x,\ u(0) = 0,\ u\left(\pi/2\right) = 0.$$

iii) Show that Chebyshev polynomial $T_n(z)$ satisfies the second order differential equation

$$(1 - z^2)\frac{d^2w}{dz^2} - z\frac{dw}{dz} + n^2 w = 0.$$ (4+3+3)

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