KANYASHREE UNIVERSITY

M.Sc. 1st Semester Examination-2024

Subject: Mathematics

Course- CC 3

Linear Algebra

Full Marks-40 **Time-2.00 Hours**

GROUP - A

(Answer **any four** of the following) **(5×4=20)**

1. Let $\alpha = (a_1, a_2, a_3)$, $\beta = (b_1, b_2, b_3) \in R^3$ and $(\alpha, \beta) = |a_1 b_1 + a_2 b_2 + a_3 b_3|$. Does (α, β) define a real inner product on R^3? Justify your answer.

2. Let V be a Euclidean space and $\alpha, \beta \in V$. Then prove that $|(\alpha, \beta)| \leq \|\alpha\|\|\beta\|$, with equality if and only if $\alpha \; and \; \beta$ are linearly dependent.

3. Let $(\alpha_1, \alpha_2, \alpha_3), (\beta_1, \beta_2)$ be ordered bases of the real vector spaces V and W respectively. A linear mapping $T: V \longrightarrow W$ maps the basis vectors as $T(\alpha_1) = \beta_1 + \beta_2$, $T(\alpha_2) = 3\beta_1 - \beta_2$ and $T(\alpha_3) = \beta_1 + 3\beta_2$. Find the matrix of T relative to the ordered bases $(\alpha_1, \alpha_2, \alpha_3)$ of V and (β_1, β_2) of W.

4. State and prove Sylvester's Law of Inertia.

5. Suppose V is a real vector space and V_C be the complexification of V. Then prove that the dimension of V_C (as a complex vector space) is same as the dimension of V (as a real vector space).

6. Prove that minimal polynomial of a square matrix is unique.

GROUP – B

(Answer **any two** of the following) **(10× 2=20)**

7. i) Let V be a finite-dimensional inner product space. Then show that V has an orthogonal basis.

   ii) Apply the Gram-Schmidt orthogonalization process to find an orthogonal basis and then an orthonormal basis for the subspace U of R^4 spanned by $v_1 = (1,1,1,1)$, $v_2 = (1,2,4,5)$, $v_3 = (1, -3, -4, -2)$. (5+5)

8. i) Define idempotent linear operator. Give an example of a linear operator T having eigen values 0 and 1 but T is not idempotent.

   ii) Give an example of a linear operator $T : R^3 \rightarrow R^3$ such that $T \neq 0$, $T^2 \neq 0$ but $T^3 = 0$.

   iii) Does there exist a linear mapping $f : R^3 \rightarrow R^3$ with the property that f(0,1, 1) = (3, 1, -2), f(1, 0, 1) = (4,-1, 1), f(1, 1, 0) = (-3,2, 1) and f(1, 1, 1) = (3,4, 2)? (4+3+3)

9. i) Find the dual basis of the basis B = {(1, -1, 3), (0, 1, -1), (0, 3, -2)} of R^3.

   ii) Determine the range of values of λ for which the quadratic form

$$\lambda(x^2 + y^2 + z^2) + 2(xy - yz + zx)$$

is positive definite. (5+5)

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