KANYASHREE UNIVERSITY

M.Sc. 1st Semester Examination-2024

Subject: Mathematics

Course- CC 2

Complex Analysis

Full Marks-40 **Time-2.00 Hours**

GROUP - A

(Answer **any four** of the following) **(5×4=20)**

1. Prove that a continuous function $w = f(z) = u(x,y) + iv(x,y)$ is differentiable in a domain $\mathcal{D}$ if the four partial derivatives $u_x(x,y)$, $u_y(x,y)$, $v_x(x,y)$ and $v_y(x,y)$ exists, are continuous and satisfying Cauchy-Riemann equations at each point of $\mathcal{D}$.
2. State and prove Cauchy's Integral formula for derivatives.
3. State and prove Liouville's theorem. Hence prove the fundamental theorem of classical algebra.
4. Show that $\cosh\left(z + \frac{1}{z}\right) = a_0 + \sum_{n=1}^{\infty} a_n(z^n + \frac{1}{z^n})$, where $a_n = \frac{1}{2\pi}\int_0^{2\pi} \cos n\theta \cosh(2\cos\theta) d\theta$.
5. Prove that the limit point of the zeros of an analytic function is an essential singularity of the function, unless the function is identically zero.
6. Find the bilinear transformation which maps the point $1, i, -1$ in the z-plane into the points $0, 1, \infty$ in the w-plane. Show that by means of this transformation the area of the circle $|z| = 1$ is represented in the w-plane by the half-plane above the real axis.

GROUP – B

(Answer **any two** of the following) **(10× 2=20)**

7. i) State and prove Cauchy's Residue Theorem and hence deduce $\oint_{|z|=2} \frac{e^z}{z(z-1)^2} dz = 2\pi i$.

   ii) Show that $\frac{a^n}{n!} = \frac{1}{2\pi i}\oint_C \frac{e^{az}}{z^{n+1}} dz$, where C is any simple closed rectifiable curve around the origin.

   (4+3)+3

8. i) State and prove Rouche's Theorem.

   ii) Prove that all the roots of the equation $z^7 - 5z^3 + 12 = 0$ lie in $1 < |z| < 2$.

   (5+5)

9. i) Evaluate: $\int_{-\infty}^{\infty} \frac{\sin mx}{x}\, dx$, $(m > 0)$.

   ii) Prove that the function f has a pole of multiplicity m at α iff in some neighbourhood of α, f can be expressed in the form $f(z) = \frac{\psi(z)}{(z-\alpha)^m}$, where ψ is analytic at α and $\psi(\alpha) \neq 0$.

   (6+4)

****