KANYASHREE UNIVERSITY

M.Sc. 1st Semester Examination-2024

Subject: Mathematics

Course- CC 1

Real Analysis

Full Marks-40 **Time-2.00 Hours**

GROUP - A

(Answer **any four** of the following) **(5×4=20)**

1. Show that if $f: [a, b] \to \mathbb{R}$ is a function of bounded variation on [a,b], then f is bounded on [a,b]. If f and g are two functions of bounded variation on [a,b], then show that $f.g$ is also of bounded variation on $[a, b]$. (2+3)

2. Show that the Cantor set C has power c. 5

3. Define addition of two cardinal numbers with justification. Show that every open interval has the power of continuum. (2+3)

4. State Carathéodory's definition of Lebesgue measure, and using this definition show that complement of a measurable set is measurable. Also, show that if outer measure of a set is zero, then it is measurable. (1+2+2)

5. Show that sum of two measurable functions is also measurable. Give the definition of simple function with example. State Lusin's theorem. (1+2+2)

6. Define Borel set with example. State and prove Monotone convergence theorem. (2+1+2)

GROUP – B

(Answer **any two** of the following) **(10× 2=20)**

7. (i) Let f be a function of bounded variation on [a,b]. If f is continuous at a point $x_0 \in [a, b]$, then show that the function $F(x) = V_f[a, x]$ is also continuous at x_0.

   (ii) Show that if f is absolutely continuous on [a,b], then so is |f|. Hence or otherwise verify whether the following function is absolutely continuous on [0,1].

$$f(x) = \begin{cases} x^2 \left|\sin \frac{1}{x}\right|, & x \neq 0 \\ 0 & , x = 0 \end{cases}$$

   (iii) Given the cardinal numbers m and n, show that $m \leq n$ if and only if there is a cardinal number p such that $n = m + p$. 3+(2+2)+3

8. (i) If E be a measurable set, then show that there exists a $\mathcal{G}_\delta$-set $G \supset E$ such that $m^*(G - E) = 0$.

   (ii) If a function f defined on E is continuous a.e., then f is measurable on E.

(iii) Compute the RS-integrals if exists: $(a) \int_0^{\pi} \cos x \; d(\sin x)$

(b) $\int_0^4 (x^2 + x + 1) \, d[x]$. 3+3+(2+2)

9. (i) Show that every Borel set in $\mathbb{R}$ is measurable.

(ii) Let f and g be bounded measurable functions defined on a set E of finite measure. If $f \leq g$ a.e., then show that $\int_E f \leq \int_E g$.

(iii) State and prove Lebesgue dominated convergence theorem. 3+2+(1+4)

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